TRINITY COLLEGE

Year 12 Mathematics Applications
Test 2 2018

SECTION 1 – CALCULATOR FREE
Sequences, Graphs and Networks

STUDENT'S NAME ____

DATE: Friday 6th April **TIME:** 25 minutes **MARKS**: 32

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Consider the sequence with the n^{th} term P_n given by $P_n = 42 - 3n$.

(a) State the first three terms [3]

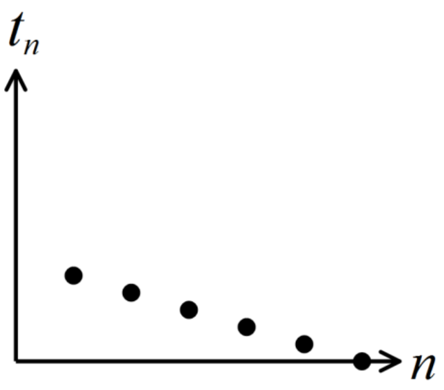
(b) State the first order recurrence relation for the sequence. [2]

(c) Is this sequence arithmetic, geometric or neither? [1]

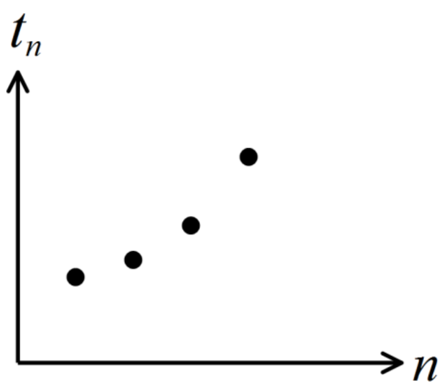
2. (6 marks)

(a) Match each of the graphs below with the most likely type of first order recurrence relation and write your answer in the table provided on the following page. [4]

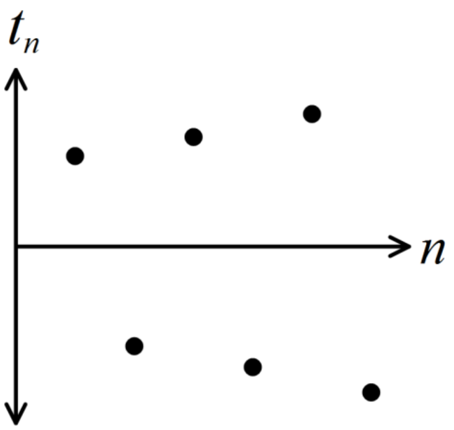
1.



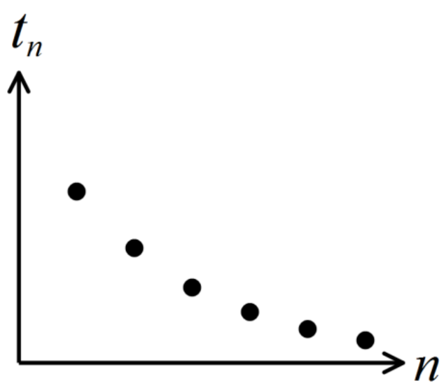
2.



3.



4.



A $t_{n+1} = t_n - 1$ $t_1 = 5$

B $t_{n+1} = -1.1t_n$ $t_1 = 5$

C $t_{n+1} = \frac{2}{3}t_n + 1$ $t_1 = 10$

D $t_{n+1} = 2t_n - 4$ $t_1 = 5$

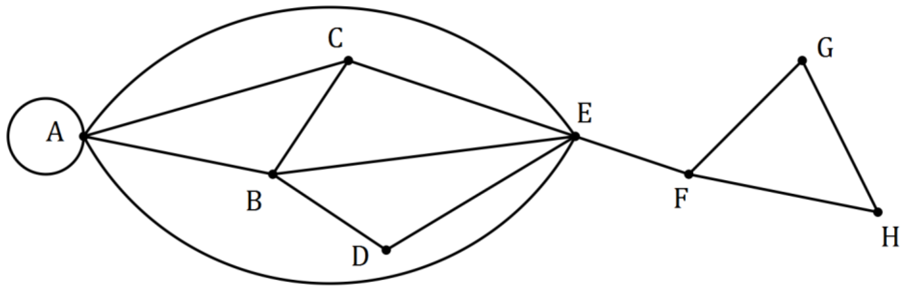
Graph	1	2	3	4
Recurrence relation				

(b) Which one of the above recurrence relations has a long-term steady state solution? Evaluate it's steady state solution.

[2]

3. (11 marks)

Todd and his friend Rob are kayaking enthusiasts. The graph below represents a number of canals that meet at locations A to H. Todd and Rob are planning to kayak through these canals.



(a) Todd argues that it would be possible to kayak a particular trail that travels through each canal only once and returns to the starting location. However, Rob argues that whilst you can kayak through each canal once, you will end up at a different location to where you started.

(i) What is the correct term for the type of trail that Todd is referring to? [1]

(ii) What is the correct term for the type of trail that Rob is referring to? [1]

(iii) Who is correct? Justify your answer. [2]

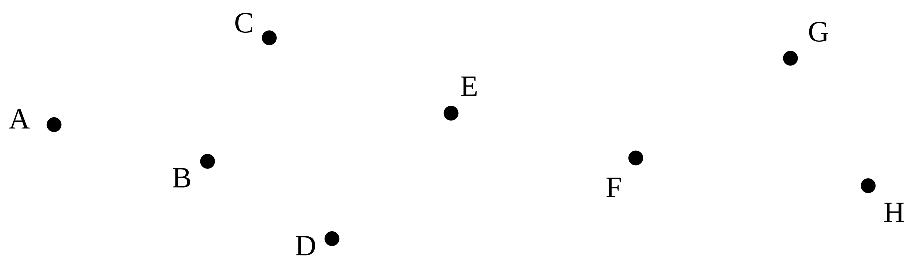
(iv) Suggest a route, starting at any location, that could be taken in order to travel through each canal just once. [1]

(b)

(i) If Todd and Rob started at location C, suggest a route they could take to visit each location just once without repreating a canal or a location. [2]

(ii) What is the correct term for this type of journey throughout a network? [1]

(c) Draw a subgraph of the above graph that is simple, connected, has no bridges and has 8 edges. [3]



4. (4 marks)

A connected planar graph has four faces and seven edges.

(a) Determine the number of vertices the graph has. [2]

(b) Sketch a graph with these properties. [2]

5. (5 marks)

Ellie, Amy, Beatrix, Christopher and Dave all enjoy playing tennis.
Christopher has played tennis with Amy, Beatrix and Ellie.
Dave played tennis with Amy and Ellie.

(a) Sketch and label a network that connects people with the people they have played tennis with.
[3]

(b) Explain why the graph is an example of a bipartite graph. [2]

TRINITY COLLEGE

Year 12 Mathematics Applications
Test 2 2018

SECTION 2 – CALCULATOR ASSUMED
Sequences, Graphs and Networks

STUDENT'S NAME ____

DATE: Friday 6th April **TIME:** 25 minutes **MARKS**: 25

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (5 marks)

Your blood alcohol content (BAC) measures the amount of alcohol you have in your system in milligrams of alcohol per 100 millilitres of blood. The BAC of a person was recorded *n* hours after the consumption of alcohol, giving the following values:

Time (n) (hours)	0	1	2	3
BAC (B) (mg/100mL)	80	48	28.8	17.28

(a) Determine a rule that represents the blood alcohol content of this person *n* hours after the consumption of alcohol. [3]

Use the rule from part (a) to answer the following questions.

(b) Calculate the BAC of the person after 5 hours. [1]

(c) The BAC is considered to be zero once it falls below one mg/100mL. During which hour, after the consumption of alcohol, will this occur? [1]

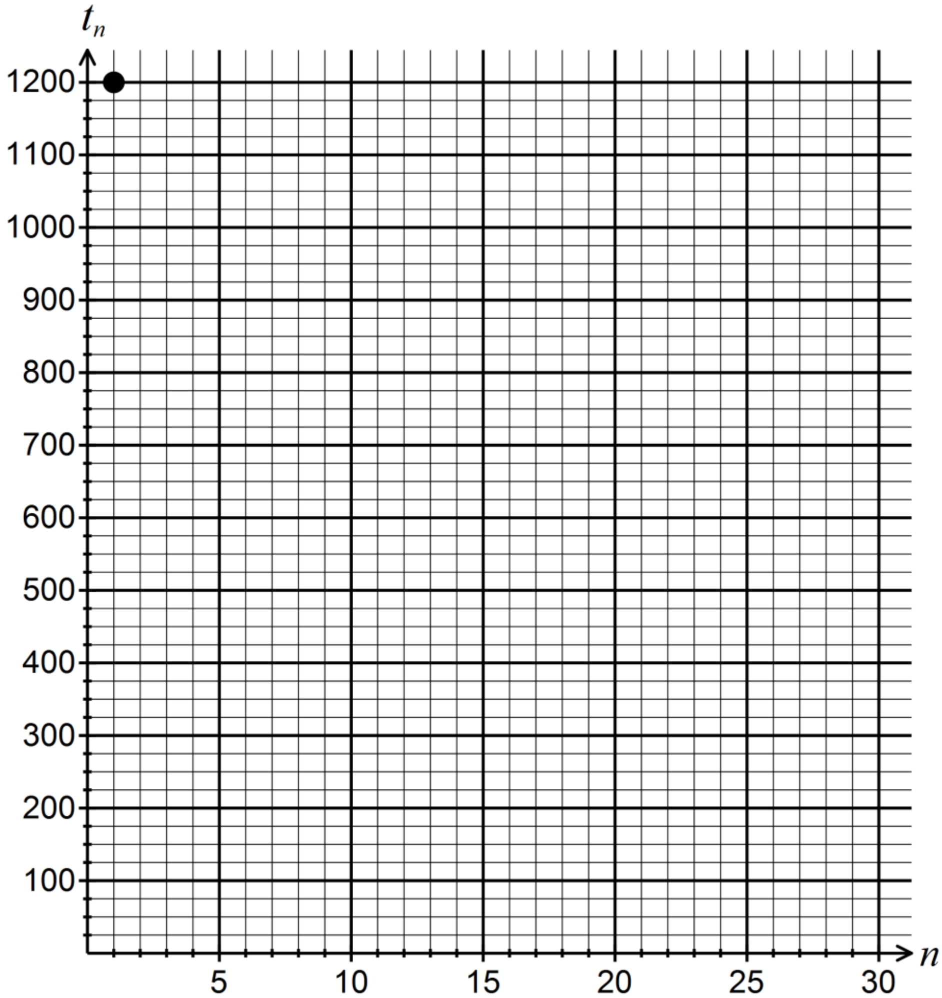
6. (10 marks)

In the Daintree rainforest in northern Queensland, the cassowary population is dropping each year. The current population is 1200. A scheme is being trialled under which 15 cassowaries are introduced to the Daintree each year.
The population of cassowaries in the Daintree can be modelled by the first order recurrence relation $T_{n+1} = 0.955T_n + b \qquad T_1 = 1200$, where T_n is the number of cassowaries in the Daintree at the beginning of the n^{th} year.

(a) Interpret the coefficient 0.955 in the context of the question. [1]

(b) State the value of b. [1]

(c) Graph the number of cassowaries in the Daintree for every fifth year (commencing at n = 5), up to the 30th year, on the axes below. [2]



(d) Using your graph, comment on how the population of cassowaries is changing over time. [1]

(e) To the nearest whole number, what is the long-term effect on the cassowary population? [2]

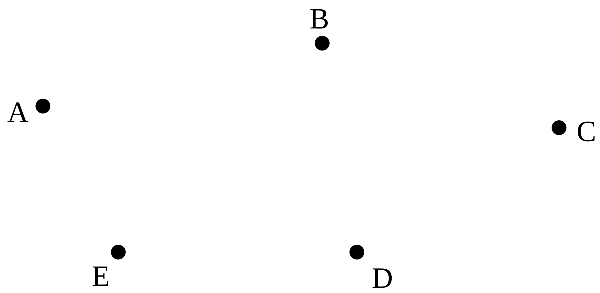
(f) The Wildlife Management Unit are aiming to maintain the current population of cassowaries. How many cassowaries would need to be introduced to the Daintree each year to achieve this goal? [3]

7. (6 marks)

Consider the following adjacency matrix M

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 1 & 0 \\ 2 & 0 & 2 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(a) Draw the graph that corresponds to this adjacency matrix [3]



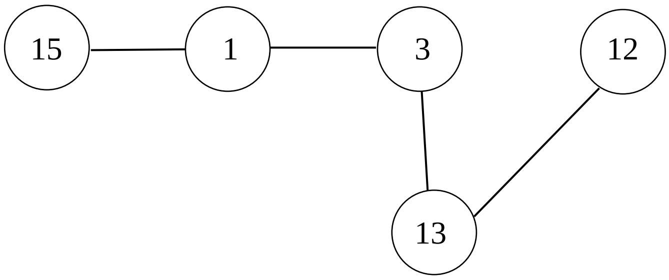
(b) What feature of the graph can be explained by the row and column of zeros? [1]

(c) How many walks of length three are there from A to D? [2]

8. (4 marks)

The first fifteen integers: 1, 2, 3,…15, can be written in a path (or sequence) such that the sum of every adjacent pair of integers is a square number.

E.g. A sample of five of these integers could be written (or connected) in sequence as shown below:



Determine the entire path (or sequence) for each of the first fifteen integers, that satisfies this condition.